

QCD thermodynamics at phenomenologically relevant coupling

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References:

PRL 104 (2010) 122003; JHEP 08 (2010) 113; PLB 696 (2011) 468; JHEP 08 (2011) 053;
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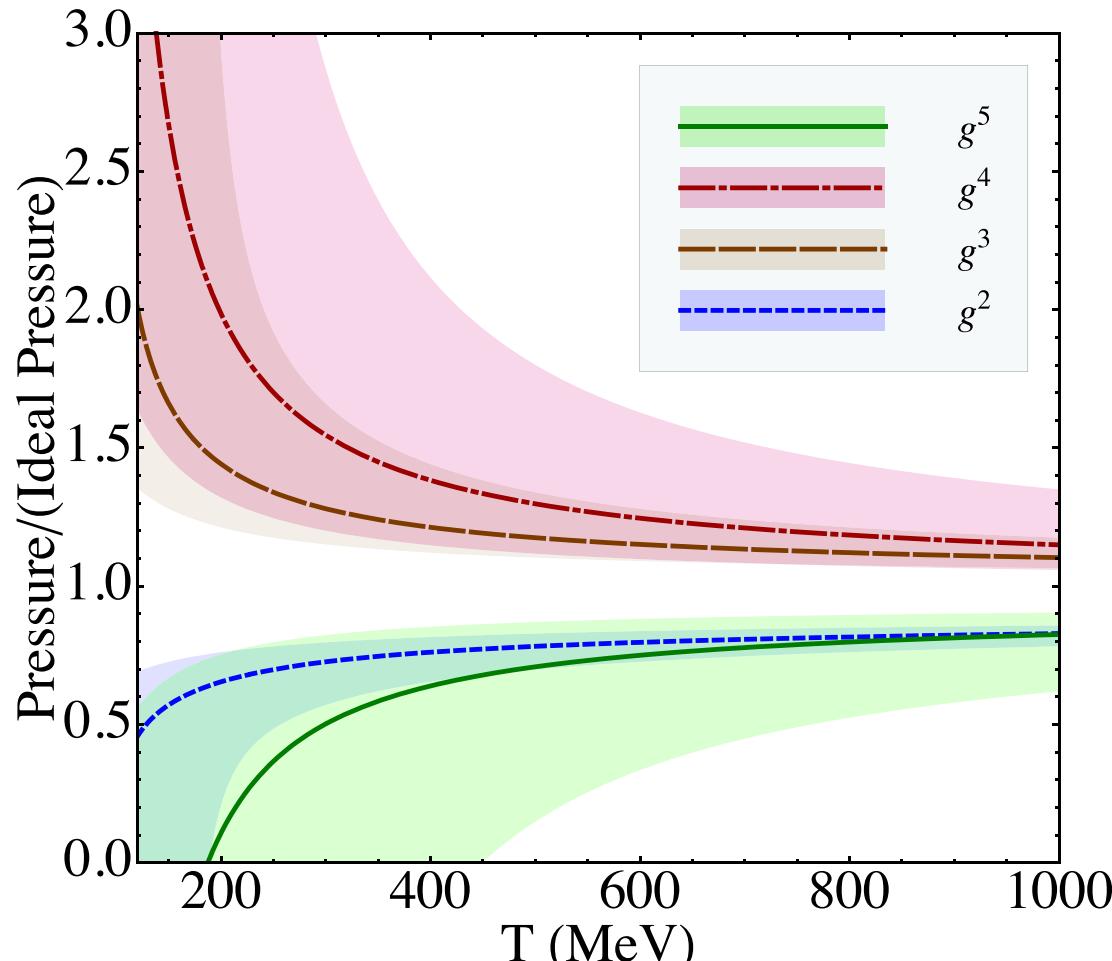
HENPIC, 19/06/14

Universität Bielefeld

Introduction – Heavy ion collisions $\rightarrow g \sim \mathcal{O}(1)$

- RHIC: $T_0 \sim 2 T_c$; LHC: $T_0 \sim 4 T_c$ ($T_c \simeq 160$ MeV)
- Quark Gluon Plasma (QGP) vs Quark Gluon Liquid (QGL)
- Running coupling expected: $g \sim \mathcal{O}(1)$
 - Phenomenologically relevant coupling
 - Neither tiny, nor huge: **INTERMEDIATE!**
 - Conventional perturbation theory: $g \ll 1$ (scale separation)
 - Strong-coupling formalisms: $g \gg 1$
- How to tackle it directly using (resummed) perturbation theory?!

Nonconvergence of canonical thermal QCD



Weak-coupling expansion QCD free energy with $N_c = 3$ and $N_f = 3$ vs temperature.

$$(\pi T \leq \Lambda \leq 4\pi T \text{ & } \alpha_s = g^2/4\pi)$$

- Weak-coupling expansion of QCD free energy (pressure) known to order $g^6 \log g$ (partial 4-loop)^{1,2,3,4,5,6,7}
- Typical scale: $2\pi T$ (central curves)
- At RHIC and LHC temperatures, running coupling $\alpha_s \sim 0.3$ or $g \sim 2$
- Successive terms can only strictly form a decreasing series if $\alpha_s \lesssim 1/20$ or $T \gtrsim 10^5$ GeV
- Small coupling \neq perturbative!
- **RESUMMATION** is needed!

¹ Shuryak, 78; ² Kapusta, 79; ³ Toimela, 85;

⁴ Arnold and Zhai, 94/95;

⁵ Kastening and Zhai, 95;

⁶ Braaten and Nieto, 96;

⁷ Kajantie, Laine, Rummukainen and Schröder, 02.

Finite temperature field theory primer

- At high temperatures, $T \gg M_{\text{bare}}$: Massless particles!
- More scales than in vacuum: Nonanalytic contributions to free energy from medium

a_0	$a_2 g^2$		$a_4 g^4$		$a_6 g^6$...	(T , hard)
		$b_3 g^3$	$b_4 g^4$	$b_5 g^5$	$b_6 g^6$...	(gT , soft, hard thermal loops)
					$c_6 g^6$...	($g^2 T$, ultrasoft, nonperturbative!)
d_0	$d_2 g^2$	$d_3 g^3$	$d_4 g^4$	$d_5 g^5$	$d_6 g^6$...	

- gT – (Chomo)Electric (Debye) mass
 $g^2 T$ – (Chromo)Magnetic mass (Linde problem!)
- Massless particles → Massive quasiparticles!
- Weak-coupling expansion expands around an ideal gas of massless particles – NOT appropriate d.o.f. at high T

Electric scale gT

(perturbative)

Hard-thermal-loop perturbation theory (HTLpt)

- A **reorganization** of thermal QCD (Andersen, Braaten and Strickland, 99)

$$\mathcal{L}_{\text{HTLpt}} = (\mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{HTL}} - \delta \mathcal{L}_{\text{HTL}}) \Big|_{g \rightarrow \sqrt{\delta} g}$$

- Hard-Thermal-Loop (HTL) effective action (Braaten and Pisarski, 92)

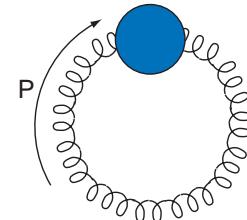
$$\mathcal{L}_{\text{HTL}} = -\frac{1}{2} m_D^2 \text{Tr} \left(G_{\mu\alpha} \left\langle \frac{y^\alpha y^\beta}{(y \cdot D)^2} \right\rangle_y G^\mu{}_\beta \right)$$

- Adding \mathcal{L}_{HTL} SHIFTS expansion to an **ideal gas of massive (Debye) quasiparticles** – Appropriate d.o.f. at high T
- δ : Expansion parameter – # of HTL dressed loops
- Other approaches: Dimensional reduction (DR) (Braaten and Nieto, 95), DR screened perturbation theory (Blaizot, Iancu and Rebhan, 99)

HTLpt – LO free energy for Yang-Mills

- Separation into hard and soft contributions ($d = 3 - 2\epsilon$)

$$\mathcal{F}_g = -\frac{1}{2} \oint_P \left\{ (d-1) \ln[-\Delta_T^{\text{HTL}}(P)] + \ln[\Delta_L^{\text{HTL}}(P)] \right\}$$



- Hard momenta ($\omega, \mathbf{p} \sim T$)

$$\begin{aligned} \mathcal{F}_g^{(h)} = & \frac{d-1}{2} \oint_P \ln P^2 + \frac{1}{2} m_D^2 \oint_P \frac{1}{P^2} - \frac{1}{4(d-1)} m_D^4 \oint_P \left[\frac{1}{P^4} \right. \\ & \left. - 2 \frac{1}{p^2 P^2} - 2d \frac{1}{p^4} \mathcal{T}_P + 2 \frac{1}{p^2 P^2} \mathcal{T}_P + d \frac{1}{p^4} (\mathcal{T}_P)^2 \right] + \mathcal{O}(m_D^6) \end{aligned}$$

- Soft momenta ($\omega, \mathbf{p} \sim gT$)

$$\mathcal{F}_g^{(s)} = \frac{1}{2} T \int_{\mathbf{p}} \ln(p^2 + m_D^2)$$

HTLpt – LO free energy for Yang-Mills

- LO thermodynamical potential

$$\frac{\Omega_{\text{LO}}}{\mathcal{F}_{\text{ideal}}} = 1 - \frac{15}{2} \hat{m}_D^2 + 30 \hat{m}_D^3 + \frac{45}{4} \left(\log \frac{\hat{\Lambda}}{2} - \frac{7}{2} + \gamma_E + \frac{\pi^2}{3} \right) \hat{m}_D^4 + \mathcal{O}(\hat{m}_D^6)$$

$$(\mathcal{F}_{\text{ideal}} \equiv -\frac{(N_c^2-1)\pi^2 T^4}{45}, \hat{x} \equiv \frac{x}{2\pi T})$$

- Mass prescriptions

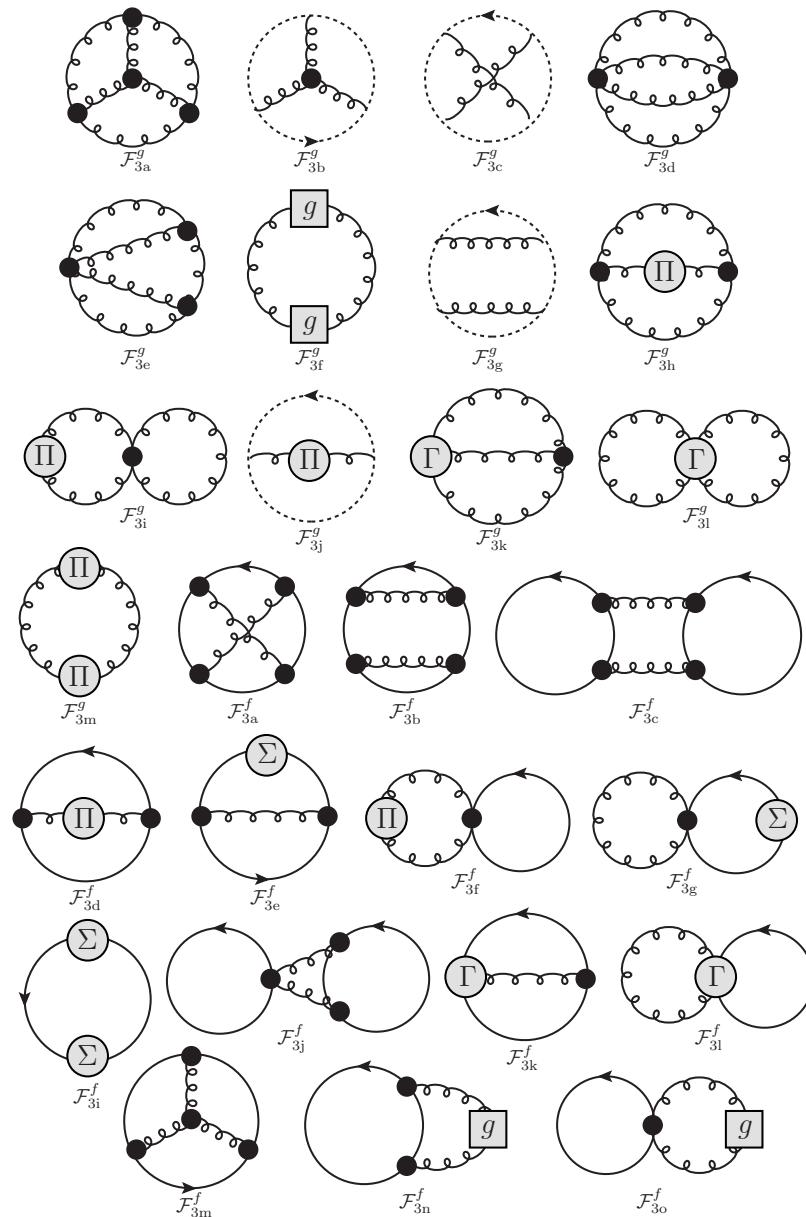
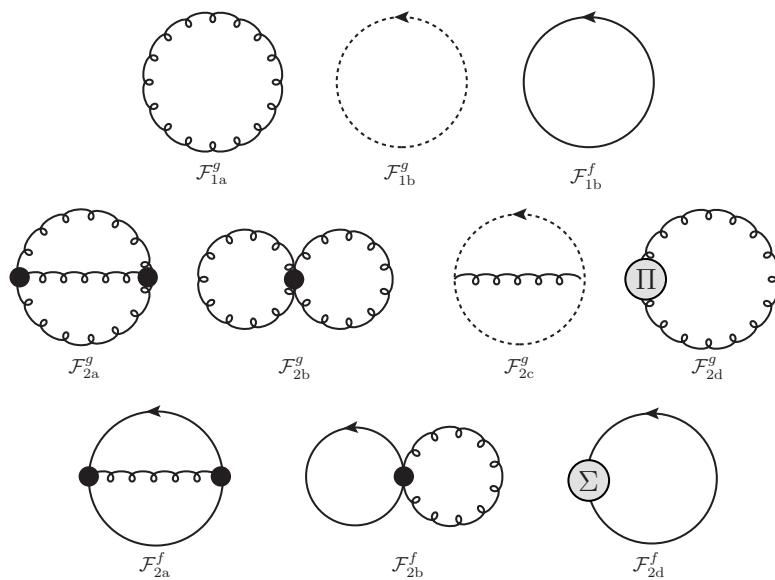
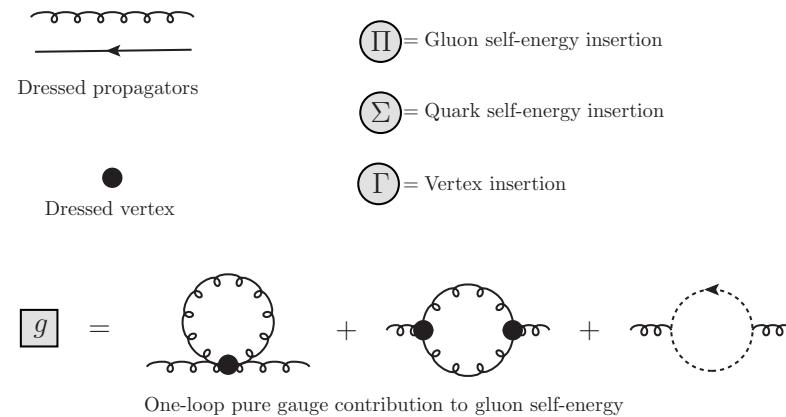
- Variational mass:

$$\frac{\partial \Omega(T, \alpha_s, m_D, \delta = 1)}{\partial m_D} = 0$$

not well-defined at LO (α_s absent!)

- Perturbative mass: $m_D = \sqrt{\frac{N_c}{3}} g T$

HTLpt – QCD diagrams through NNLO (49)



HTLpt – NNLO thermodynamic potential for QCD

- For QCD with general N_c and N_f at finite T

$$\begin{aligned}
\frac{\Omega_{\text{NNLO}}}{\mathcal{F}_{\text{ideal}}} = & 1 + \frac{7}{4} \frac{d_F}{d_A} - \frac{15}{4} \hat{m}_D^3 + \frac{c_A \alpha_s}{3\pi} \left[-\frac{15}{4} + \frac{45}{2} \hat{m}_D - \frac{135}{2} \hat{m}_D^2 - \frac{495}{4} \left(\log \frac{\hat{\Lambda}}{2} + \frac{5}{22} + \gamma_E \right) \hat{m}_D^3 \right] \\
& + \frac{s_F \alpha_s}{\pi} \left[-\frac{25}{8} + \frac{15}{2} \hat{m}_D + 15 \left(\log \frac{\hat{\Lambda}}{2} - \frac{1}{2} + \gamma_E + 2 \log 2 \right) \hat{m}_D^3 - 90 \hat{m}_q^2 \hat{m}_D \right] \\
& + \left(\frac{c_A \alpha_s}{3\pi} \right)^2 \left[\frac{45}{4} \frac{1}{\hat{m}_D} - \frac{165}{8} \left(\log \frac{\hat{\Lambda}}{2} - \frac{72}{11} \log \hat{m}_D - \frac{84}{55} - \frac{6}{11} \gamma_E - \frac{74}{11} \frac{\zeta'(-1)}{\zeta(-1)} \right. \right. \\
& \left. \left. + \frac{19}{11} \frac{\zeta'(-3)}{\zeta(-3)} \right) + \frac{1485}{4} \left(\log \frac{\hat{\Lambda}}{2} - \frac{79}{44} + \gamma_E + \log 2 - \frac{\pi^2}{11} \right) \hat{m}_D \right] \\
& + \left(\frac{c_A \alpha_s}{3\pi} \right) \left(\frac{s_F \alpha_s}{\pi} \right) \left[\frac{15}{2} \frac{1}{\hat{m}_D} - \frac{235}{16} \left(\log \frac{\hat{\Lambda}}{2} - \frac{144}{47} \log \hat{m}_D - \frac{24}{47} \gamma_E + \frac{319}{940} + \frac{111}{235} \log 2 \right. \right. \\
& \left. \left. - \frac{74}{47} \frac{\zeta'(-1)}{\zeta(-1)} + \frac{1}{47} \frac{\zeta'(-3)}{\zeta(-3)} \right) + \frac{315}{4} \left(\log \frac{\hat{\Lambda}}{2} - \frac{8}{7} \log 2 + \gamma_E + \frac{9}{14} \right) \hat{m}_D + 90 \frac{\hat{m}_q^2}{\hat{m}_D} \right] \\
& + \left(\frac{s_F \alpha_s}{\pi} \right)^2 \left[\frac{5}{4} \frac{1}{\hat{m}_D} + \frac{25}{12} \left(\log \frac{\hat{\Lambda}}{2} + \frac{1}{20} + \frac{3}{5} \gamma_E - \frac{66}{25} \log 2 + \frac{4}{5} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{2}{5} \frac{\zeta'(-3)}{\zeta(-3)} \right) \right. \\
& \left. - 15 \left(\log \frac{\hat{\Lambda}}{2} - \frac{1}{2} + \gamma_E + 2 \log 2 \right) \hat{m}_D + 30 \frac{\hat{m}_q^2}{\hat{m}_D} \right] + s_{2F} \left(\frac{\alpha_s}{\pi} \right)^2 \left[\frac{15}{64} (35 - 32 \log 2) - \frac{45}{2} \hat{m}_D \right]
\end{aligned}$$

PURELY ANALYTIC!!!

HTLpt – Mass prescriptions for QCD

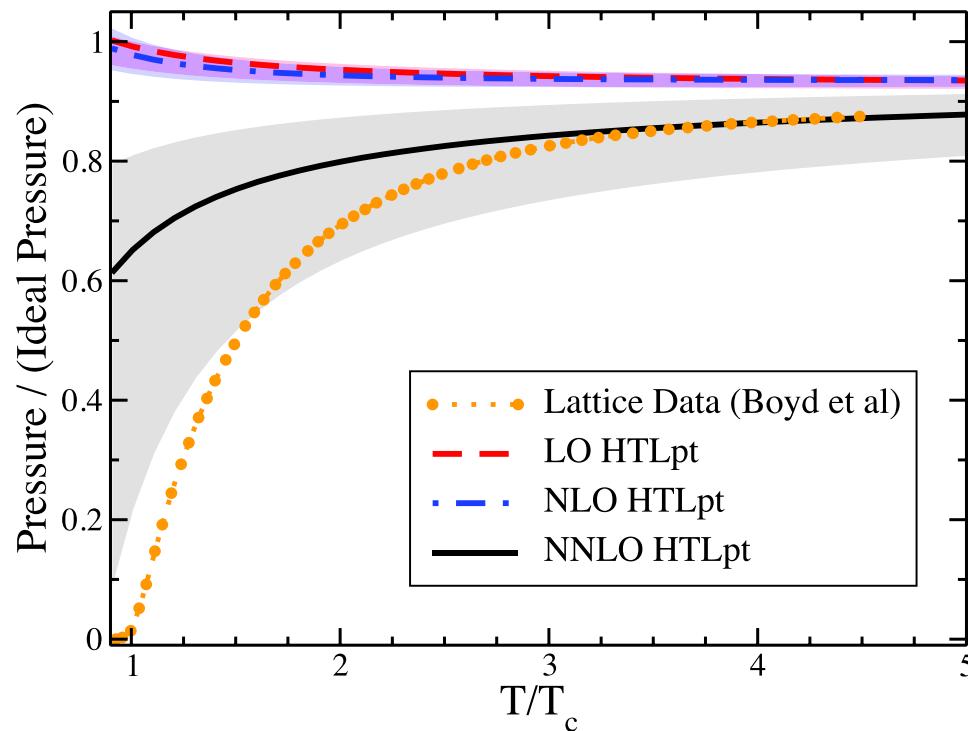
- Gauge-invariant NLO electric mass from DR (Braaten and Nieto, 96)

$$\begin{aligned} m_D^2 = & \frac{4\pi\alpha_s}{3} T^2 \left\{ c_A + s_F + \frac{c_A^2\alpha_s}{3\pi} \left(\frac{5}{4} + \frac{11}{2}\gamma_E + \frac{11}{2}\log\frac{\hat{\Lambda}}{2} \right) \right. \\ & + \frac{c_A s_F \alpha_s}{\pi} \left(\frac{3}{4} - \frac{4}{3}\log 2 + \frac{7}{6}\gamma_E + \frac{7}{6}\log\frac{\hat{\Lambda}}{2} \right) \\ & \left. + \frac{s_F^2\alpha_s}{\pi} \left(\frac{1}{3} - \frac{4}{3}\log 2 - \frac{2}{3}\gamma_E - \frac{2}{3}\log\frac{\hat{\Lambda}}{2} \right) - \frac{3}{2} \frac{s_F\alpha_s}{\pi} \right\} \end{aligned}$$

- $m_q = 0$ from NNLO variational gap equation

HTLpt – Yang-Mills and QCD pressures $\mathcal{P} = -\mathcal{F}$

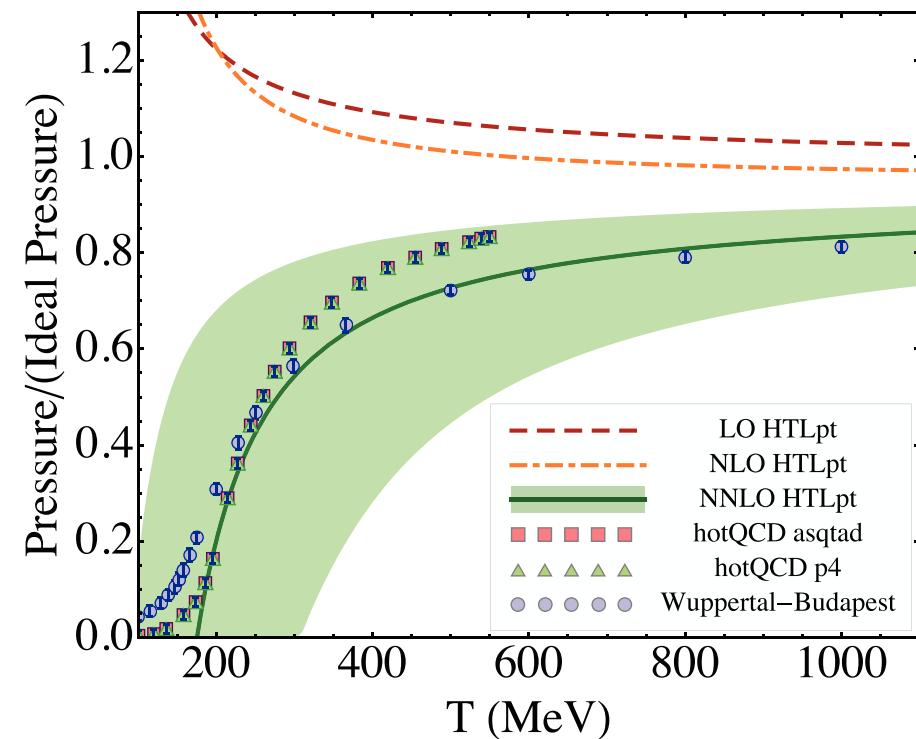
Yang-Mills



Andersen, Strickland and NS,

PRL 104 (2010) 122003 & JHEP 08 (2010) 113

QCD with $N_f = 3$

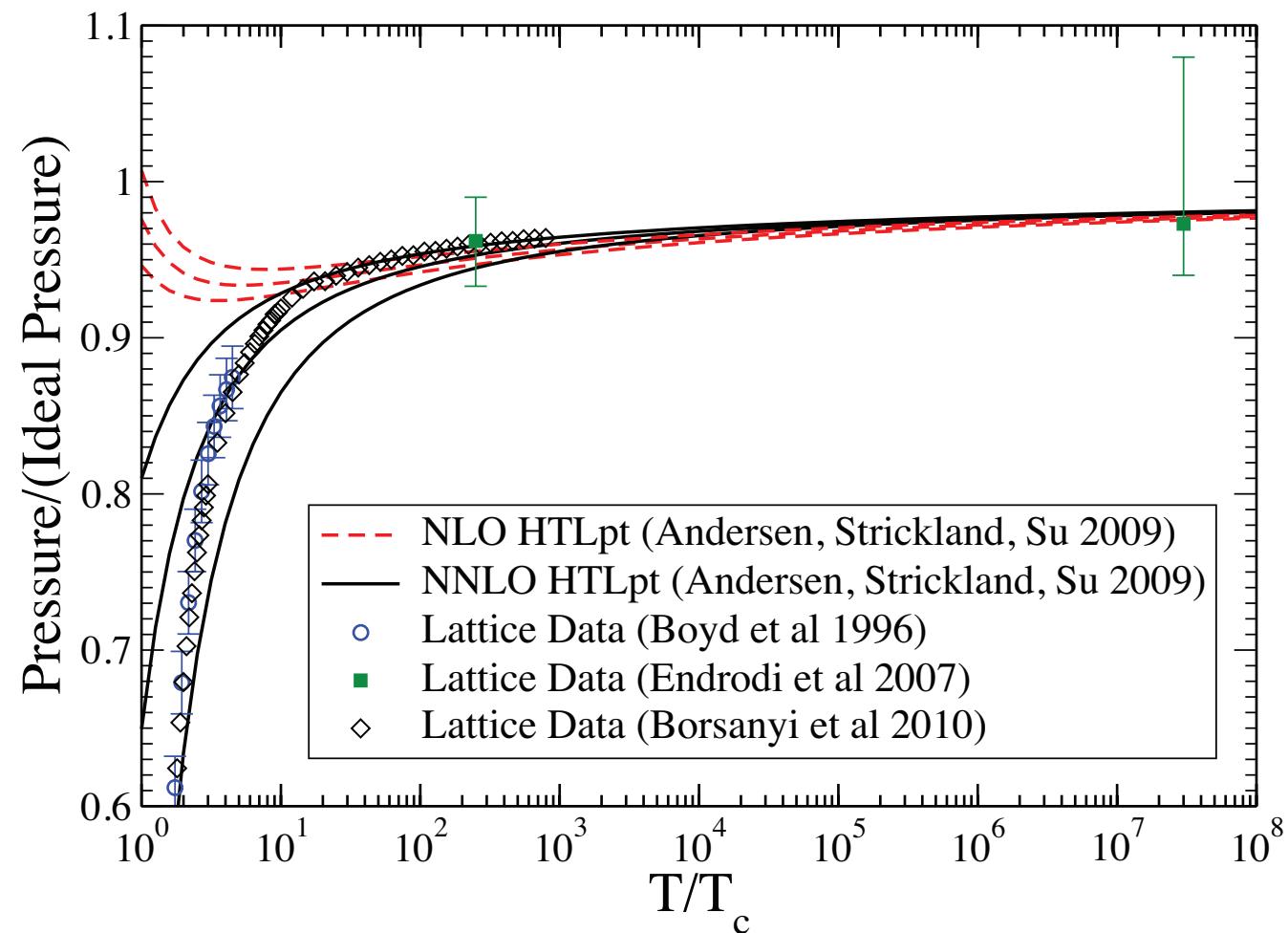


Andersen, Leganger, Strickland and NS,

PLB 696 (2011) 468 & JHEP 08 (2011) 053

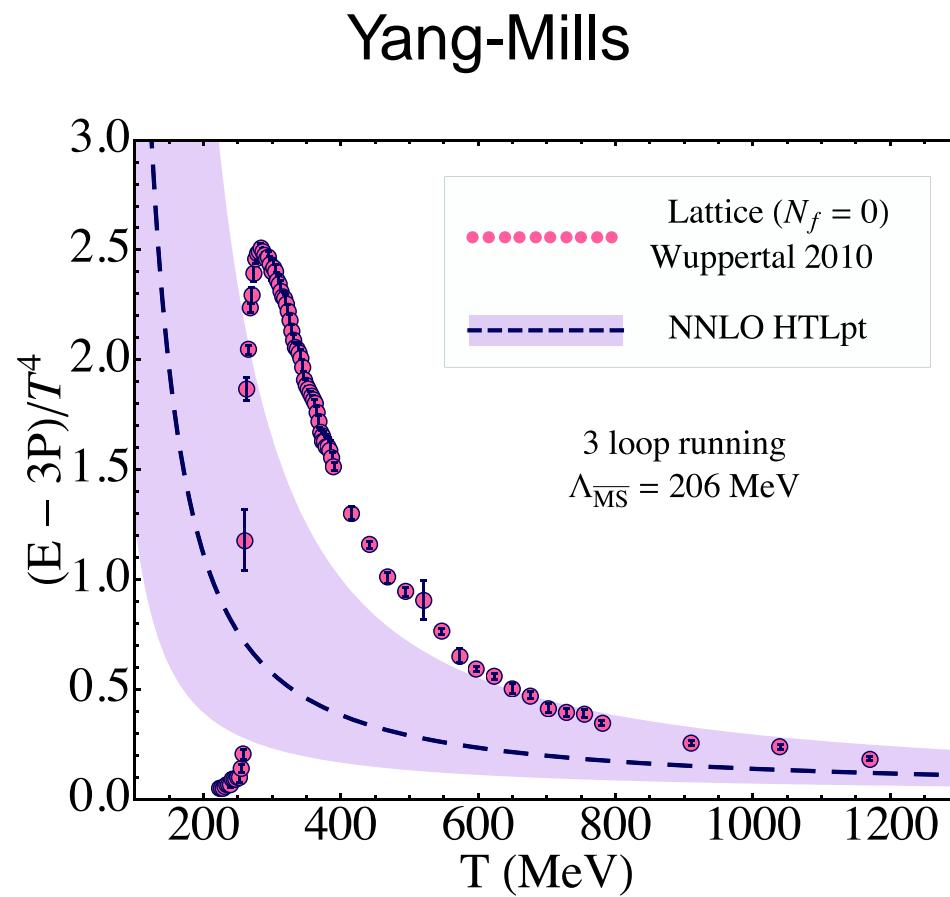
HTLpt – Yang-Mills high T pressure

Improvement of convergence: $\sim 10^5 T_c \rightarrow \sim 10^1 T_c$

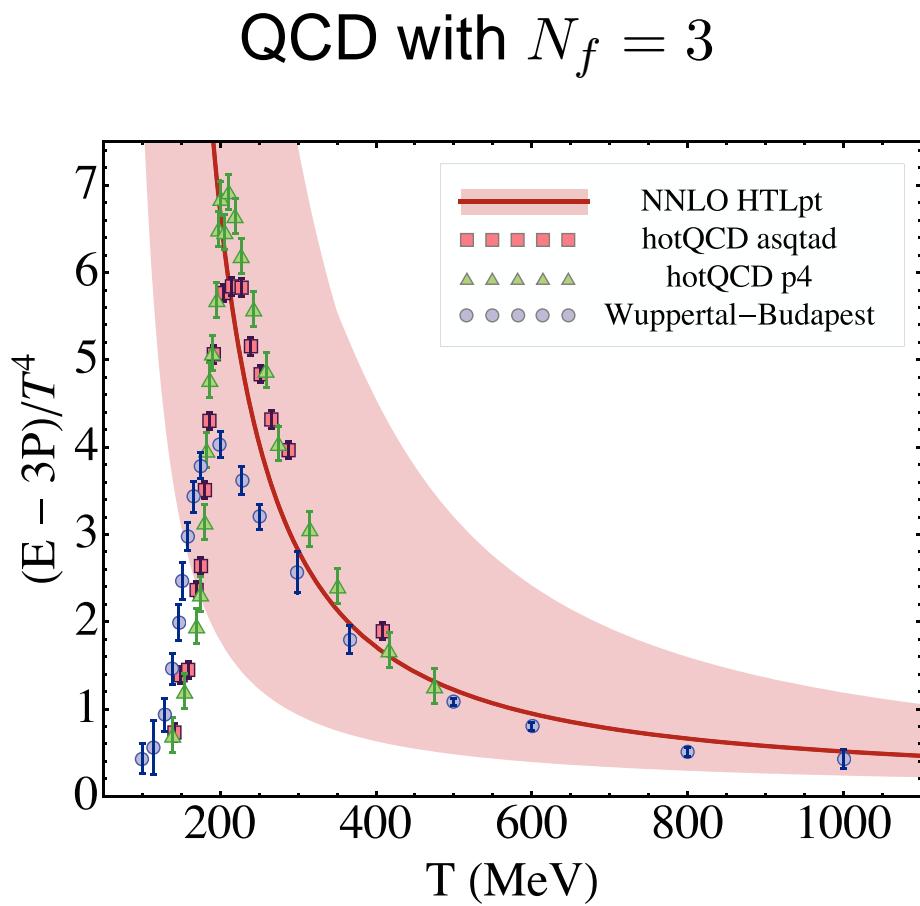


Andersen, Strickland and NS, JHEP 08 (2010) 113

HTLpt – Yang-Mills and QCD trace anomalies



Andersen, Strickland and NS, JHEP 1008, 113 (2010)



Andersen, Leganger, Strickland and NS,
PLB 696 (2011) 468 & JHEP 08 (2011) 053

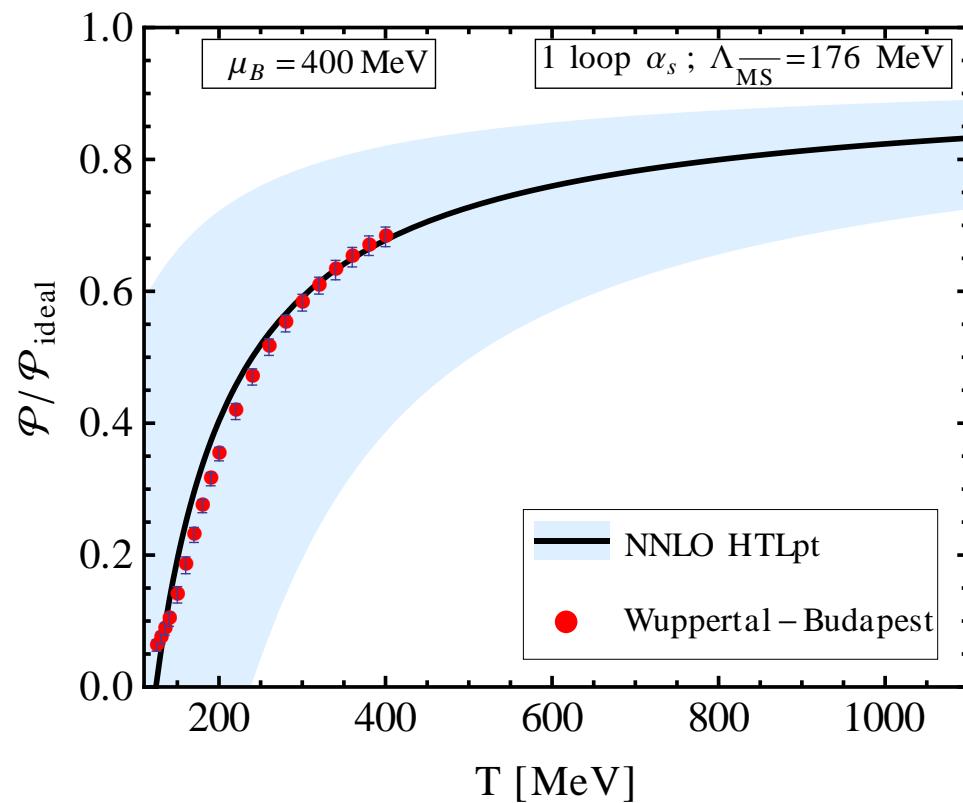
HTLpt – Ω_{NNLO} at finite T and μ (analytic!)

$$\begin{aligned}
\frac{\Omega_{\text{NNLO}}}{\Omega_0} = & \frac{7}{4} \frac{d_F}{d_A} \frac{1}{N_f} \sum_f \left(1 + \frac{120}{7} \hat{\mu}_f^2 + \frac{240}{7} \hat{\mu}_f^4 \right) - \frac{s_F \alpha_s}{\pi} \frac{1}{N_f} \sum_f \left[\frac{5}{8} (1 + 12\hat{\mu}_f^2) (5 + 12\hat{\mu}_f^2) - \frac{15}{2} (1 + 12\hat{\mu}_f^2) \hat{m}_D \right. \\
& - \frac{15}{2} \left(2 \ln \frac{\hat{\Lambda}}{2} - 1 - \aleph(z_f) \right) \hat{m}_D^3 + 90 \hat{m}_q^2 \hat{m}_D \left. \right] + \frac{s_{2F}}{N_f} \left(\frac{\alpha_s}{\pi} \right)^2 \sum_f \left[\frac{15}{64} \left\{ 35 - 32 (1 - 12\hat{\mu}_f^2) \frac{\zeta'(-1)}{\zeta(-1)} + 472 \hat{\mu}_f^2 \right. \right. \\
& + 1328 \hat{\mu}_f^4 + 64 \left(-36 i \hat{\mu}_f \aleph(2, z_f) + 6(1 + 8\hat{\mu}_f^2) \aleph(1, z_f) + 3i \hat{\mu}_f (1 + 4\hat{\mu}_f^2) \aleph(0, z_f) \right) \left. \right\} - \frac{45}{2} \hat{m}_D (1 + 12\hat{\mu}_f^2) \left. \right] \\
& + \left(\frac{s_F \alpha_s}{\pi} \right)^2 \frac{1}{N_f} \sum_f \frac{5}{16} \left[96 (1 + 12\hat{\mu}_f^2) \frac{\hat{m}_q^2}{\hat{m}_D} + \frac{4}{3} (1 + 12\hat{\mu}_f^2) (5 + 12\hat{\mu}_f^2) \ln \frac{\hat{\Lambda}}{2} + \frac{1}{3} + 4\gamma_E + 8(7 + 12\gamma_E) \hat{\mu}_f^2 \right. \\
& + 112 \hat{\mu}_f^4 - \frac{64 \zeta'(-3)}{15 \zeta(-3)} - \frac{32}{3} (1 + 12\hat{\mu}_f^2) \frac{\zeta'(-1)}{\zeta(-1)} - 96 \left\{ 8 \aleph(3, z_f) + 12 i \hat{\mu}_f \aleph(2, z_f) - 2(1 + 2\hat{\mu}_f^2) \aleph(1, z_f) - i \hat{\mu}_f \aleph(0, z_f) \right\} \left. \right] \\
& + \left(\frac{s_F \alpha_s}{\pi} \right)^2 \frac{1}{N_f^2} \sum_{f,g} \left[\frac{5}{4 \hat{m}_D} (1 + 12\hat{\mu}_f^2) (1 + 12\hat{\mu}_g^2) + 90 \left\{ 2(1 + \gamma_E) \hat{\mu}_f^2 \hat{\mu}_g^2 - \left\{ \aleph(3, z_f + z_g) + \aleph(3, z_f + z_g^*) \right. \right. \right. \\
& + 4i \hat{\mu}_f [\aleph(2, z_f + z_g) + \aleph(2, z_f + z_g^*)] - 4 \hat{\mu}_g^2 \aleph(1, z_f) - (\hat{\mu}_f + \hat{\mu}_g)^2 \aleph(1, z_f + z_g) - (\hat{\mu}_f - \hat{\mu}_g)^2 \aleph(1, z_f + z_g^*) \\
& \left. \left. \left. - 4i \hat{\mu}_f \hat{\mu}_g^2 \aleph(0, z_f) \right\} \right\} - \frac{15}{2} (1 + 12\hat{\mu}_f^2) (2 - 1 - \aleph(z_g)) \hat{m}_D \right] + \left(\frac{c_A \alpha_s}{3\pi} \right) \left(\frac{s_F \alpha_s}{\pi N_f} \right) \sum_f \left[\frac{15}{2 \hat{m}_D} (1 + 12\hat{\mu}_f^2) \right. \\
& - \frac{235}{16} \left\{ \left(1 + \frac{792}{47} \hat{\mu}_f^2 + \frac{1584}{47} \hat{\mu}_f^4 \right) \ln \frac{\hat{\Lambda}}{2} - \frac{144}{47} (1 + 12\hat{\mu}_f^2) \ln \hat{m}_D + \frac{319}{940} \left(1 + \frac{2040}{319} \hat{\mu}_f^2 + \frac{38640}{319} \hat{\mu}_f^4 \right) - \frac{24\gamma_E}{47} (1 + 12\hat{\mu}_f^2) \right. \\
& - \frac{44}{47} \left(1 + \frac{156}{11} \hat{\mu}_f^2 \right) \frac{\zeta'(-1)}{\zeta(-1)} - \frac{268}{235} \frac{\zeta'(-3)}{\zeta(-3)} - \frac{72}{47} [4i \hat{\mu}_f \aleph(0, z_f) + (5 - 92\hat{\mu}_f^2) \aleph(1, z_f) + 144i \hat{\mu}_f \aleph(2, z_f) + 52 \aleph(3, z_f)] \left. \right\} \\
& + 90 \frac{\hat{m}_q^2}{\hat{m}_D} + \frac{315}{4} \left\{ \left(1 + \frac{132}{7} \hat{\mu}_f^2 \right) + \frac{11}{7} (1 + 12\hat{\mu}_f^2) \gamma_E + \frac{9}{14} \left(1 + \frac{132}{9} \hat{\mu}_f^2 \right) + \frac{2}{7} \aleph(z_f) \right\} \hat{m}_D \left. \right] + \frac{\Omega_{\text{NNLO}}^{\text{YM}}}{\Omega_0}
\end{aligned}$$

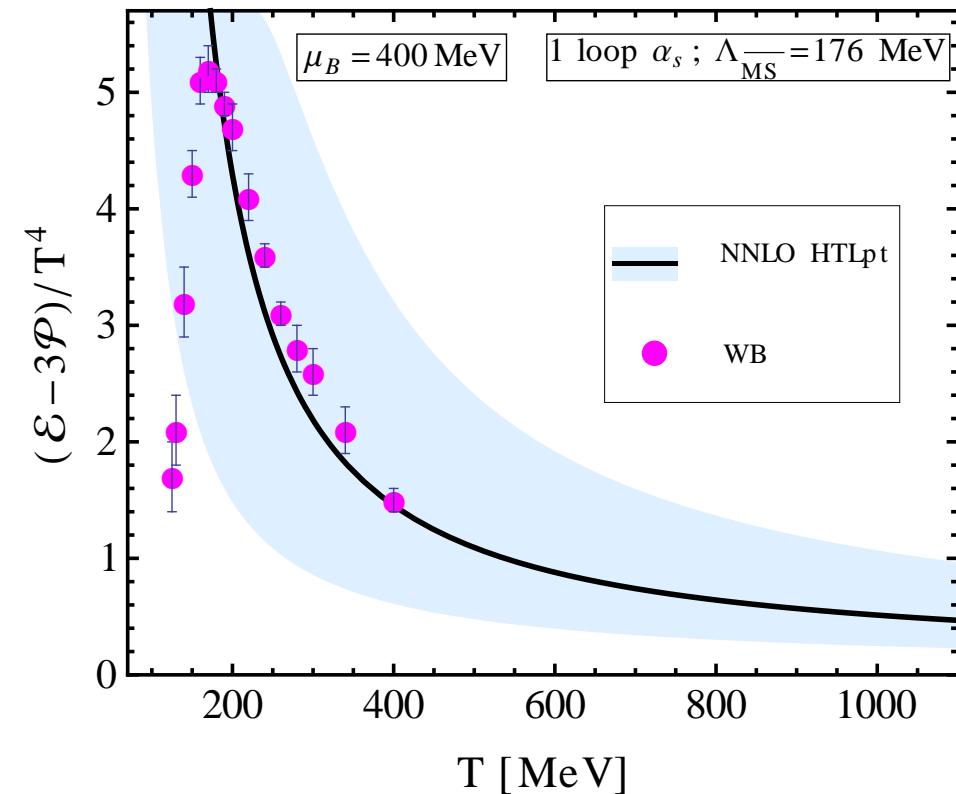
Haque, Andersen, Mustafa, Strickland and NS, PRD 89 (2014) 061701;
Haque, Bandyopadhyay, Andersen, Mustafa, Strickland and NS, JHEP 05 (2014) 027

HTLpt – QCD EoS at finite T and μ

Pressure



Trace anomaly



$$(\Lambda = \Lambda_q = 2\pi\sqrt{T^2 + \mu^2/\pi^2}, \Lambda_g = 2\pi T)$$

Haque, Andersen, Mustafa, Strickland and NS, PRD 89 (2014) 061701;
 Haque, Bandyopadhyay, Andersen, Mustafa, Strickland and NS, JHEP 05 (2014) 027

Quark(Baryon) number susceptibilities

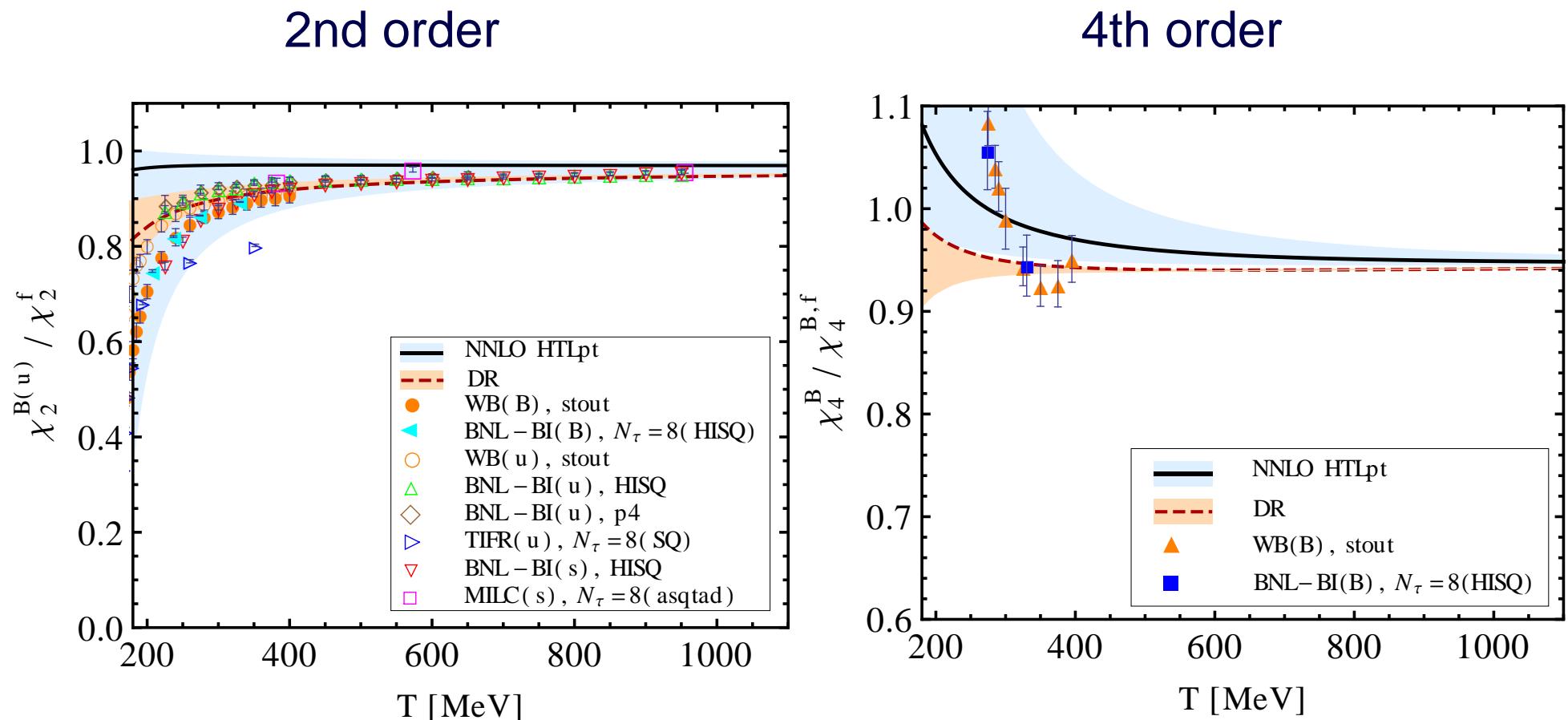
- Responses of the system to density fluctuations
- Important measures for deconfinement transition
- Quark number susceptibilities, $\mu \equiv (\mu_u, \mu_d, \dots, \mu_{N_f})$

$$\chi_{ijk\dots}(T) \equiv - \left. \frac{\partial^{i+j+k+\dots} \Omega(T, \mu)}{\partial \mu_u^i \partial \mu_d^j \partial \mu_s^k \dots} \right|_{\mu=0}$$

- Baryon number susceptibilities, $\mu_u = \mu_d = \mu_s = \mu = \frac{1}{3}\mu_B$

$$\chi_B^n(T) \equiv - \left. \frac{\partial^n \Omega}{\partial \mu_B^n} \right|_{\mu_B=0}$$

Baryon number susceptibilities: HTLpt vs DR



Mogliacci, Andersen, Strickland, NS and Vuorinen, JHEP 12 (2013) 055;

Haque, Andersen, Mustafa, Strickland and NS, PRD 89 (2014) 061701;

Haque, Bandyopadhyay, Andersen, Mustafa, Strickland and NS, JHEP 05 (2014) 027

Magnetic scale $g^2 T$ (nonperturbative)

Linde problem – Magnetic catastrophe in (resummed) PT!

(Linde, 80; Gross, Pisarski, Yaffe, 81)

- Contribution of zero Matsubara mode to vacuum diagrams with N 4-g vertices in free energy

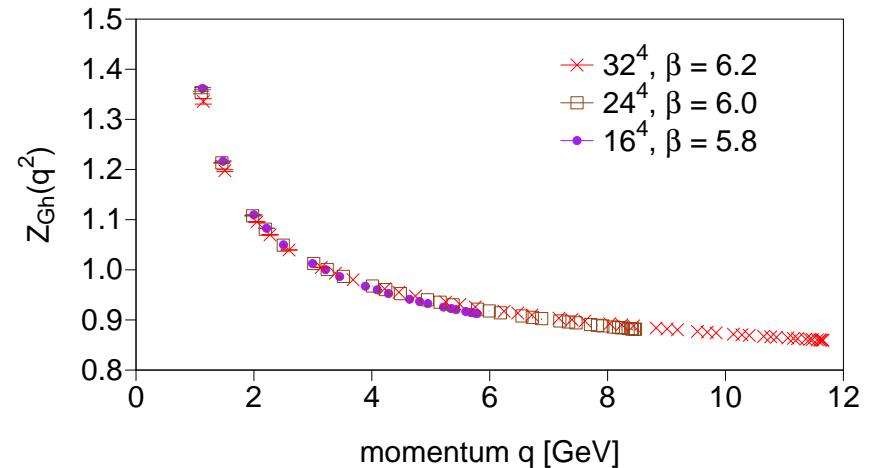
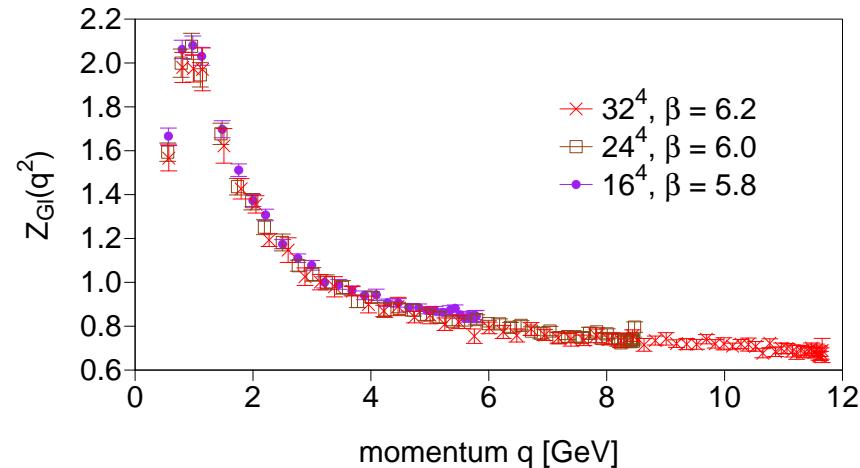
$$\begin{aligned} &\sim g^{2N} \left[T \int_p \frac{1}{p^2 + \Lambda^2} \right]^2 \left[T \int_p \frac{1}{(p^2 + \Lambda^2)^2} \right]^{N-1} \\ &\sim g^{2N} (T\Lambda)^2 \left(\frac{T}{\Lambda} \right)^{N-1} \\ &\sim g^6 T^4 \left(\frac{g^2 T}{\Lambda} \right)^{N-3}, \quad \Lambda : \text{IR cutoff} \end{aligned}$$

- $\Pi^{00(L)}(0, \mathbf{p} \rightarrow 0) \sim g^2 T^2$: Screening of static electric fields generates the scale gT (**soft**) – Perturbative
- $\Pi^{ij(T)}(0, \mathbf{p} \rightarrow 0) = 0$: Absence of Λ greater than $g^2 T$ (**ultrasoft**) for static magnetic fields – Nonperturbative!

Missing ingredients in (resummed) PT!

- Magnetic scale $g^2 T$ – Linde problem (PT breaks down at 4 loops)
- IR gauge fixing: Gluons **SUPPRESSED**; Ghosts **ENHANCED**

Features of confinement (Landau gauge)



Ilgenfritz *et al.*, PRD 83 (2011) 054506

- Topological d.o.f.: Polyakov loops, monopoles, instantons...
- **NO SURPRISE** of (resummed) PT deviating from lattice near T_c !

Gribov copies / Gribov-Singer ambiguity (Gribov, 78; Singer, 78)

- For Yang-Mills theory, residue gauge transformations (Gribov copies) in IR after FP – Nonperturbative gauge fixing in IR!
- Similar in string theory: After the 1st gauge condition, a 2nd one is needed to kill residue gauge transformations
- Functional integral should be restricted to Gribov region

$$\Omega \equiv \{ A : \partial_i A_i = 0, \text{ and } -D_i(A)\partial_i \geq 0 \}$$

- With Gribov parameter γ_G which breaks conformal symmetry,

$$E_G(\mathbf{p}) = \sqrt{\mathbf{p}^2 + \frac{\gamma_G^4}{\mathbf{p}^2}} \quad (\text{Gribov dispersion relation})$$

- Reduction of physical state space in IR as a feature of confinement mechanism (Gribov, 78; Feynman, 81; Zwanziger, 97)

Propagators from Gribov quantization in Landau gauge

Gribov quantization has **confinement** features built in:

- Gluons are **IR suppressed**

$$D_A = \langle A_\mu^a(P) A_\nu^b(-P) \rangle = \delta^{ab} \frac{P^2}{P^4 + \gamma_G^4} \left(\delta_{\mu\nu} - \frac{P_\mu P_\nu}{P^2} \right)$$

- Ghosts are **IR enhanced**

$$D_c = \langle \bar{c}^a(P) c^b(-P) \rangle = \frac{\delta^{ab}}{P^2} \frac{1}{1 - \sigma(P)} \approx \frac{\delta^{ab}}{P^4} \frac{128\pi\gamma_G^2}{N_c g^2} \quad (P \rightarrow 0)$$

- In line with lattice and functional methods!

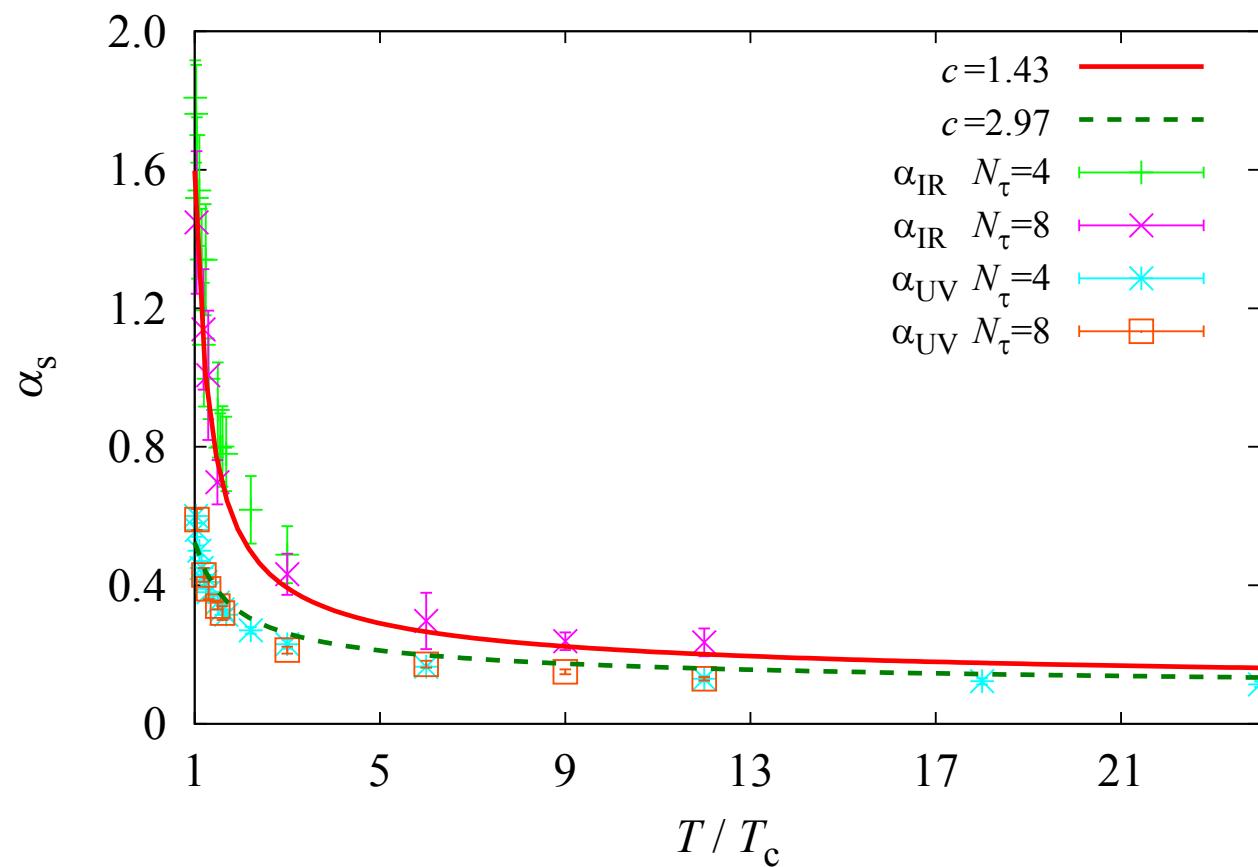
- Gap equation for γ_G : $\frac{d}{d+1} N_c g^2 \int_P \frac{1}{P^4 + \gamma_G^4} = 1$

- $T = 0$: $\gamma_G^2 = \Lambda^2 \exp \left(\frac{5}{6} - \frac{64\pi^2}{3N_c g^2} \right)$ (Gribov, 78)

- $T \rightarrow \infty$: $\gamma_G \rightarrow \frac{d}{d+1} \frac{N_c}{4\sqrt{2}\pi} g^2 T$ (Zwanziger, 07; Fukushima and NS, 13)

Running coupling near T_c

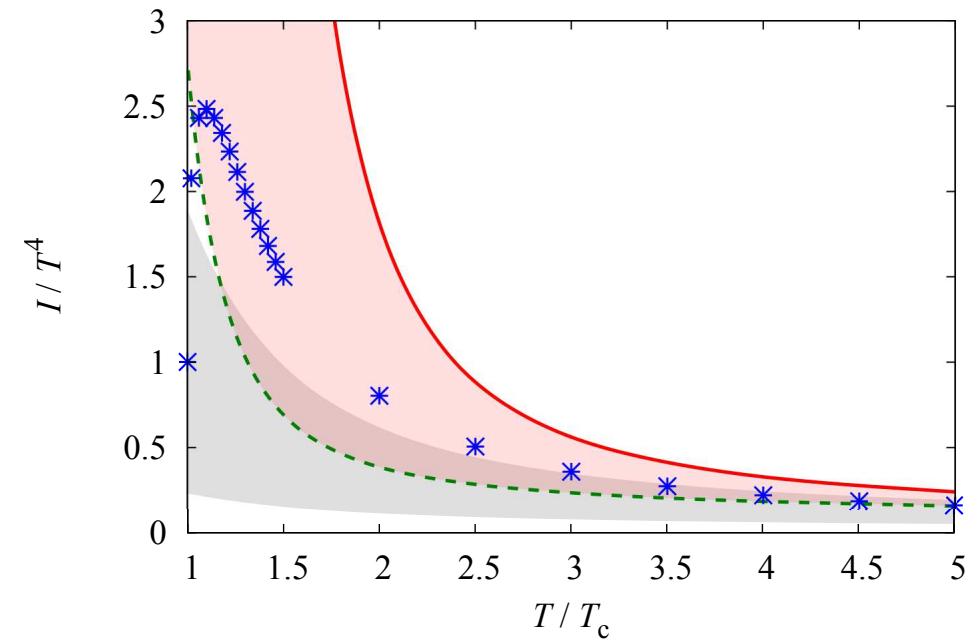
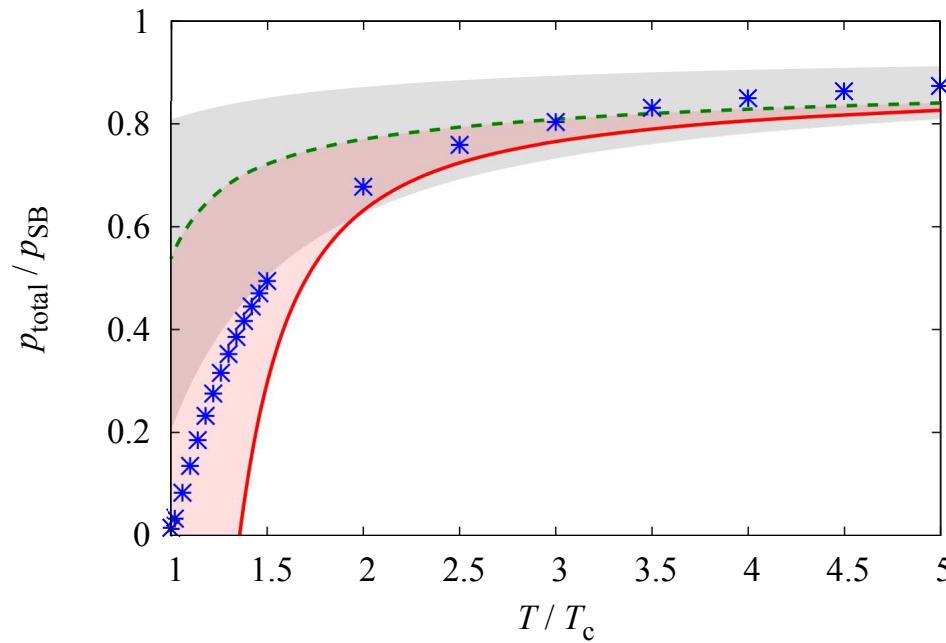
- Perturbative running **INAPPROPRIATE** near T_c !
- $\alpha_s(T)$ from lattice



Kaczmarek, Karsch, Zantow and Petreczky, PRD 70 (2004) 074505

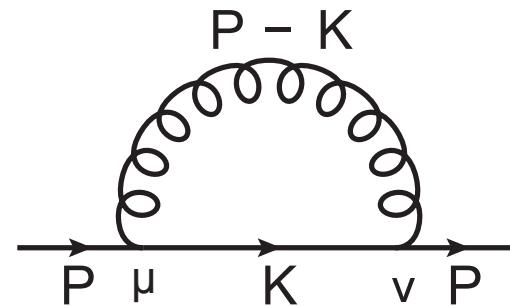
Yang-Mills EoS from Gribov quantization

Fukushima and NS, PRD 88 (2013) 076008



- $\gamma_G \sim 0.5 \text{ GeV}$; $\Lambda (= 1.69 \text{ GeV})$ fixed, uncertainty from $\alpha_s^{\text{lattice}}(T)$
- Lattice uncertainty highly suppressed above $2.5 T_c$, ROBUST!
- Good agreement with lattice, sizable contributions from $\gamma_G \sim g^2 T$ near T_c – CRUCIAL for deconfined phase!
- FRG running consistent with lattice, no qualitative change

Quark thermal mass w. Gribov gluons (NS and Tywoniuk, forthcoming)



- Quark self-energy – NO mass scale (except T)!

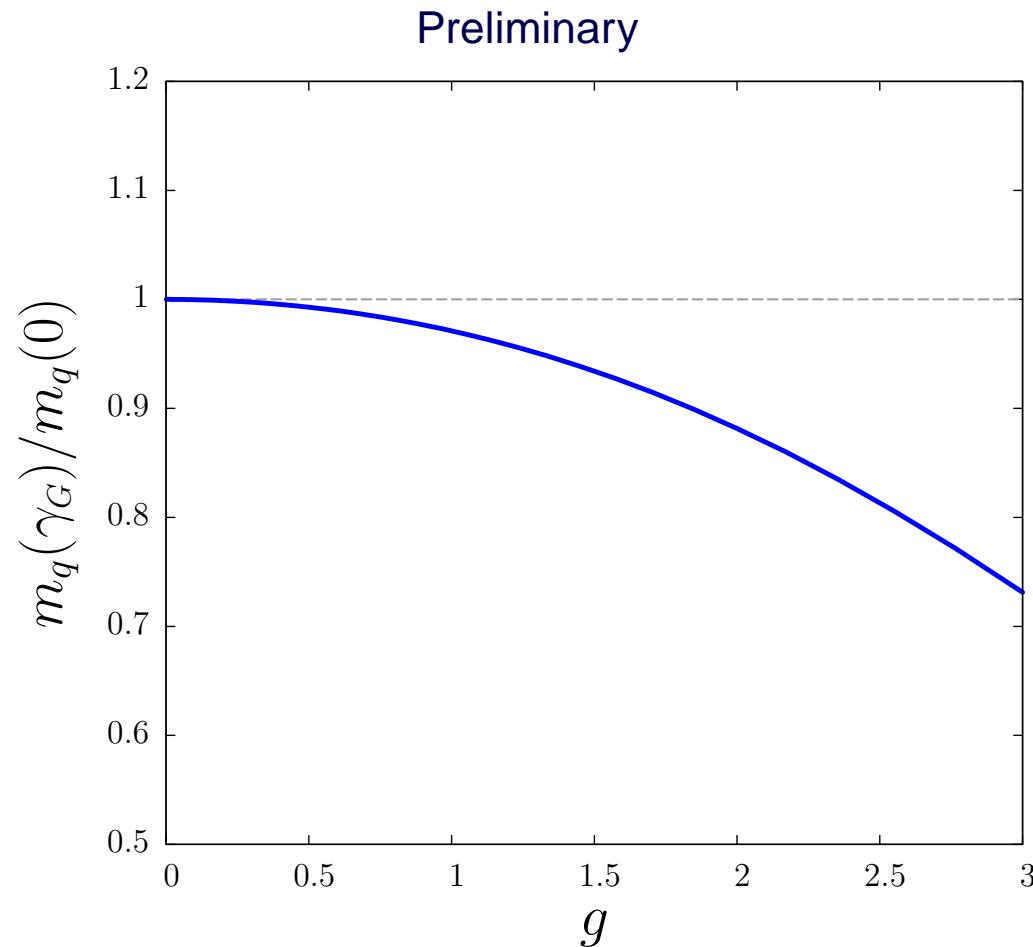
$$\Sigma(P) = (ig)^2 C_F \sum_{\{K\}} \gamma^\mu S(K) \gamma^\nu D^{\mu\nu}(P - K)$$

- IR properties of gluons are crucial in accessing full $\Sigma(P)$

$$D(P) \sim \frac{1}{P^2} \rightarrow D(P) \sim \frac{1}{P^2 + \frac{\gamma_G^4}{P^2}}$$

- Quark thermal mass: $\Sigma(P) \sim m_q^2 f(P_0, \mathbf{p})$

Quark thermal mass w. Gribov gluons (NS and Tywoniuk, forthcoming)



- Anti-screening from $g^2 T$: $m_q^2 = C_F \frac{(gT)^2}{8} [1 - 4.35972 \times 10^{-6} g^8]$
- Qualitatively in line with m_D from lattice (Kaczmarek, Karsch, Zantow and Petreczky, PRD 70 (2004) 074505; Kaczmarek and Zantow, PRD 71 (2005) 114510)

Conclusions

- QCD thermodynamics at phenomenologically relevant coupling
- Collective scales have to be incorporated appropriately in thermal QCD: Resummed PT might be able to tackle strong coupling!
- For gT , HTLpt results are **completely analytic** and in very good agreement with lattice down to $2 - 3 T_c$ especially considering there is **NO** fit parameters
- Hope gained that application of HTLpt to QGP (e.g. plasma instabilities, jet energy loss, quarkonium suppressions) is not misguided!
- For g^2T , Gribov quantization demonstrates **confinement** effects are crucial for the **deconfined phase** especially near T_c (from EoS results comparing to lattice)
- Nonperturbative gauge fixing is necessary even at finite T !

Outlook

- Application of HDLpt (zero T large μ analog of HTLpt) to the physics of compact stars
- Generalized/Nonperturbative HTL with both scales gT and $g^2T(?)$
- Impacts on realtime dynamics and phenomenology:
 - What does it mean if it COSTS energy to excite gluons in IR?
 - Long-range confining interaction
 - Antiscreening – strongly coupled!
 - Transport coefficients: η , ζ , \hat{q} , κ ...
 - Deep IR properties of QGP far-from-equilibrium evolution
- Created intense magnetic fields in HIC: $e|B| \gtrsim \Lambda_{\text{QCD}}^2$ or T_c^2
 - Gluons (both dynamical and nonperturbative) are CRUCIAL
(Kojo and NS, PLB 720 (2013) 192; PLB 726 (2013) 839)
 - Impacts on QGP evolution